Developing an Optimal Controller for Energy Minimization of an Electric Car

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***Abstract*—Designing energy-efficient optimal controllers for vehicle control is an essential step towards achieving a ubiquitous, environmentally-friendly autonomous transportation model. This study explores the design and implementation of optimal control strategies that minimize the energy used by an electric car traversing a pre-established roadway. The explored control strategies dictate the vehicle's output driving force as a function of its position on the chosen roadway through the minimization of a highly non-linear cost function that is representative of total energy use. The cost function is formulated subject to speed constraints (due to road speed limits), power limitations (due to physical limitations in the motor) and non-linear system dynamics that describe the vehicle's speed as a function of driving force and road position. The study investigates the efficacy of several control strategies based on Dynamic Programming (DP) and Model Predictive Control (MPC) approaches and compares the respective results. This study finally analyzes the feasibility and pragmatism of implementing the above control approaches in various real-world scenarios. A video summarizing this study can be found here.**

I. Background

With the recent developments in sensor-based technology coinciding with the consumer demand for cars that require less human touch, the design and creation of autonomous systems has been a major focus of the automotive industry. It is estimated that over $80B has already been invested in autonomous cars to date, with spending expected to increase exponentially over the next decade [1]. Traffic congestion alone costs Americans 6.9 billion hours and $160 billion annually. The (even miniscule) increases in vehicle efficiency that autonomous cars would provide through improved driving profiles offers the potential for great economic and time-saving benefits for society [2]. The need to develop strategies that control cars in an energy-efficient manner is further important in the context of reducing dangerous greenhouse gas emissions.

In this study, the driving force of a fully electric vehicle is controlled in an effort to minimize the vehicle's energy consumption over a pre-established pathway in Berkeley, CA. Several control strategies based on DP and MPC were implemented on the simulated system, and the strengths and weaknesses of each control approach were analyzed.

II. System Dynamics and Problem Formulation

This study models vehicle dynamics using a two-dimensional approach that represents the electric car as a point-mass for model approximation. It is assumed that the developed model perfectly describes the system dynamics. Process noise and exogenous disturbances are neglected and only the forces that are explicitly stated are considered in the model. A fully electric vehicle was chosen for this study (as opposed to an ICE car) to avoid the necessity of including changing gear reduction in the system model.

The system model was developed using many of the concepts described in [3] and the 2017 Midterm in the ME231A/EE220B class at UC Berkeley. Figure 1 shows a free-body diagram that explains the major forces acting on the proposed electric vehicle. The vehicle is subject to some dissipative aerodynamic forces F_{air}, rolling resistance F_{roll}, and resistance due to the horizontal component of gravity F_{g_x}. The vehicle is propelled forward by the electric motor, which provides F_{drive}. The system model is below. Note that m is the vehicle mass, v is the speed, p is the vehicle position, k is the position index, and θ represents the slope in the vehicle's path.

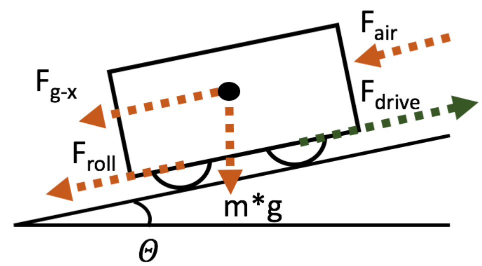

Fig. 1: Free-Body Diagram of Vehicle

$$
\begin{aligned}
m\dot{v} &= F_{\text{drive}} - \left(F_{\text{air}} + F_{\text{roll}} + F_{g_x}\right) \\
v_k &= \frac{p_{k+1} - p_k}{\Delta t} = \frac{\Delta P}{\Delta t} \\
v_{k+1} &= v_k + \Delta t \cdot \left(\frac{\Sigma F}{M}\right) \qquad (1)
\end{aligned}
$$

In order to minimize the amount of energy consumed by the car as it travels across the roadway, $F_{\text{drive}} \times \Delta P$ of the vehicle must be minimized. A cost function that reflects this goal, $J(v_0)$ along with the system constraints, is defined below. Note that asterisks denote the optimal cost and F_{air}, F_{roll}, and F_{g_x} are written in terms of their speed, mass and position components.

$$
\begin{aligned}
&J^*(v_0) = \\
&\min_{F_{\text{drive}}} \sum_{p=0}^{N-1} \left(\frac{F_{\text{drive}} + |F_{\text{drive}}|}{2} + \left(|F_{\text{drive}}| - F_{\text{drive}}\right) \right) \frac{\Delta P}{\eta(F_{\text{drive}}, v_k)} \\
&\quad \text{s.t.} \quad v_{k+1} = v_k + \frac{\Delta P}{v_k}\left(\frac{\Sigma F}{m}\right) \\
&\qquad\quad \Sigma F = A - Bv_k - Cv_k^2 - mg(\sin\theta) + F_{\text{drive}} \\
&\qquad\quad \Delta P = p_{k+1} - p_k \\
&\qquad\quad v_{k=0} = v_0 \\
&\qquad\quad F_{\min} \le F_{\text{drive}} \le F_{\max} \\
&\qquad\quad v_{\min} \le v_k \le v_{\max} \qquad (2)
\end{aligned}
$$

The system state is the speed, v, which evolves non-linearly as a function of position and forces acting on the vehicle. The system input (and decision variable) is the input driving force F_{drive}. The length of the control horizon, N, corresponds to the length of the path over which the vehicle will travel. Further note that A, B, and C are vehicle-specific constants that are derived from road load testings. F_{min} and F_{max} reflect physical limitations of the system that arise from electric motor constraints and v_{min} and v_{max} reflect the speed limits of sections of the chosen pathway.

Since we are interested in minimizing the amount of energy input (and not output), it is necessary to divide the output power by the powertrain efficiency (i.e., ratio of power at the wheels to power input from battery, η) in the cost calculations. Because η is a highly non-linear function of v_k and $F_{\text{drive},k}$, the cost function is highly non-linear.

III. Optimal Control Approaches

Both DP and MPC approaches were investigated to control vehicle speed as a function of position in order to minimize energy consumption across the chosen path. Note that the chosen path (Ashby BART Station in Berkeley, CA to the Fung Institute at UC Berkeley) was qualitatively determined to represent an "average" driving route in Berkeley, CA. The chosen route is shown below in Figure 2.

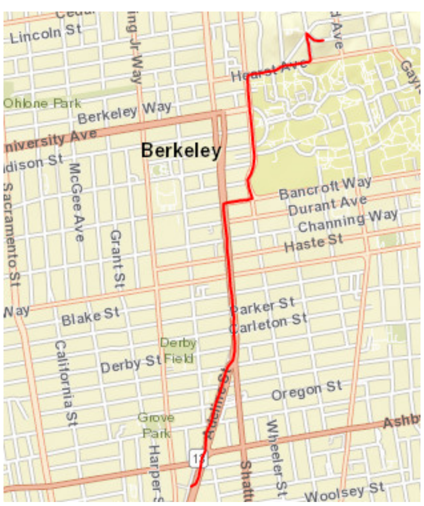

Fig. 2: Chosen Vehicle Path for Control Strategy Implementation

The DP approach involves discretizing the state (i.e., speed, v), input (i.e., driving force F_{drive}), and position spaces to generate a look-up table that correlates the measured speed at a given position to the optimal input. The MPC approach first solves an optimal control problem over a chosen prediction horizon given a measured speed at a position. The solution to this optimal control problem is a sequence of optimal inputs over the prediction horizon. The first input in this sequence is then applied to the system and the new state of the system becomes the initial state for the next optimal control problem. This pattern is then repeated over the entire length of the chosen roadway (i.e., the optimal control problem is solved recursively).

The highly non-linear nature of the powertrain efficiency function, η, presents serious challenges for implementing MPC in the system of interest. For reference, the powertrain efficiency function was derived from a map of the car's motor efficiency (Fig. 3). The powertrain efficiency is the product of the motor efficiency and the drivetrain efficiency (which is assumed to be constant). Although several methods

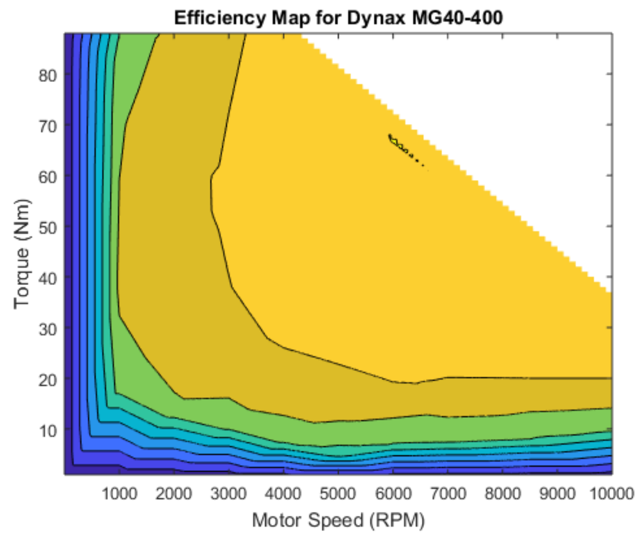

Fig. 3: Power Efficiency
Note that the torque is directly related to the F_{drive}

of calculating powertrain efficiency as a function of speed and driving force were attempted (including creating three-dimensional look-up tables and up to fourth order polynomial fits), none were compatible with the nonlinear optimization solvers with which we are familiar (e.g., fmincon, IPOPT in a YALMIP environment). We did attempt to linearize the cost function via Taylor expansion (using a polynomial fit for the efficiency function). However, analysis of the curve indicated that linearization would lead to highly inaccurate results. It was additionally unclear which equilibrium input values should be used for the Taylor expansion. Lastly, we tried to implement an MPC control strategy that used the solution to the optimal control problem with $\eta = 1$ as the initial guess for using fmincon to solve an optimal control problem that included the efficiency function. However, the fmincon solver was unable to handle this objective function format as well.

As a result, the following control strategies were explored and implemented on the chosen vehicle pathway mentioned earlier: (1) DP with the efficiency function, (2) DP assuming $\eta = 1$, and (3) MPC assuming $\eta = 1$. The results are discussed in the next section.

IV. Results

The optimal input and state trajectories for the three scenarios outlined above are shown in the top two graphs in Figure 4 below. The bottom graph demonstrates the changes in slope as a function of position along the chosen pathway.

Scenario #1 (DP control approach that properly accounts for efficiency) yields expected results, as the speed almost

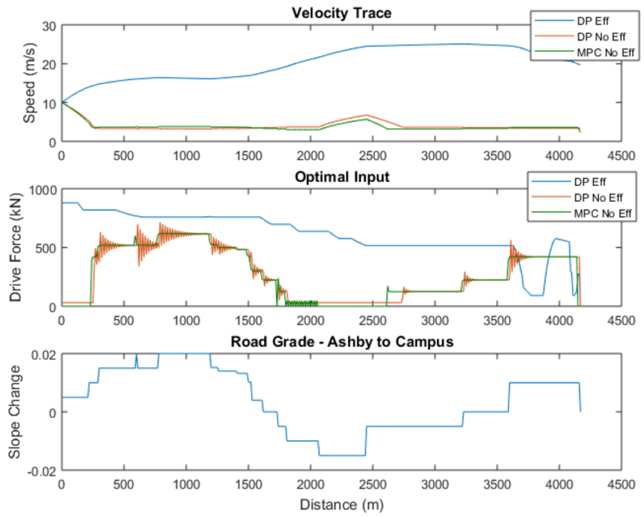

Fig. 4: Optimal State Trajectory, Optimal Input Trajectory, and Slope Along Chosen Vehicle Path

inversely scales with slope, and the upper constraints on speed limits are easily met. Scenarios #2-3 on the other hand (i.e., MPC and DP assuming $\eta = 1$) demonstrate that incorporating the powertrain efficiency function described earlier significantly changes the optimal speed profile. As seen from Fig. 2, lower input torque and lower motor speeds yield lower efficiency values. As a result, low speeds and low driving force do not necessarily imply minimum cost because efficiency penalizes the cost if the driving force and the speed are low. Physically, this means that going too slow may not be the best way to save energy.

The comparison between cost with and without powertrain efficiency also supports this intuition. When efficiency is not included, the best thing to do is to apply no input until the speed decelerates to minimum speed. Then the vehicle is maintained at the minimum speed unless a downward slope allows an increase in speed without increasing the input force. On the other hand, with the incorporation of powertrain efficiency, the input is higher and makes much more sense.

V. Discussion

This case study illustrates one of the primary advantages of DP-based control approaches over MPC-based ones: DP frameworks can handle highly non-linear constraints and cost functions. The optimization tools required to implement MPC on the other hand, are sensitive to the problem formulation. Based on this case study alone, one could erroneously conclude that a DP control strategy is more appropriate for energy minimization of an autonomous car. However, a real-world controller would likely have more states than just the speed of the car (e.g., car acceleration, distance to nearest car, among many others) and more inputs than just the driving force (e.g., steering angle). Because DP suffers from the curse of dimensionality, which implies that the computational complexity of the problem scales exponentially with the number of states and inputs, a DP-based optimal control strategy for a real-world multi-state multi-input system is much more likely to be computationally intractable than an MPC-based strategy for the same system.

A real-world version of the controller described in this study must also be able to account for unmeasured, exogenous noise. Because DP-based control strategies are finite state machines, they will likely not be able to properly account for extreme exogenous disturbances in the same way that an MPC-based approach (with online optimization) would. As a result, although DP significantly outperformed MPC in this study, a real-world implementation of this control strategy is likely only feasible with an MPC-based approach.

VI. Future Work and Acknowledgments

The question now becomes: how can an MPC control strategy be implemented to overcome the highly nonlinear cost functions and nonlinear equality constraints in a real-world, multi-state, multi-input version of this system? We will explore implementing MPC-based control strategies using more advanced nonlinear solvers in environments that are less of a "blackbox" than YALMIP (e.g., CasADI). This methodology would likely be based off of direct single-shooting, direct multiple-shooting, or direct collocation optimization methods. We will further explore MPC-based control strategies that account for mismatch between the system model and the actual system dynamics, process and measurement noise, exogenous disturbances, and incomplete state information. Such strategies would likely fall under the categories of either "Robust" or "Stochastic" MPC to ensure constraint satisfaction and would involve the use of Kalman filtering for state estimation.

We would further like to expand on this study by incorporating additional path roadways that have more extreme slope gradients and harsher dynamic speed constraints. This can simulate cars that must come to stops in extremely short distances and times, such as to avoid a collision. We would also like to quantify how much controllers designed to minimize energy consumption reduce greenhouse gas emissions in comparison with controllers that do not have environmentally-conscious objectives. To do this, we would change our cost function to minimize the amount of time to reach the final destination (or to another desirable objective that is likely to yield high energy use) and compare the energy implications to our environmentally-conscious controller accordingly. We finally thank Professors Borrelli and Packard for their time and efforts guiding us through this class.

References

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